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COMPLEX IDENTIFICATION OF THERMOPHYSICAL PROPERTIES OF ANISOTROPIC COMPOSITE MATERIAL

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Abstract – The statement and solution algorithm of a two-dimensional thermal conductivity inverse problem on complex identification of thermophysical properties of anisotropic complex material is presented. An experimental facility for carrying out thermophysical research has been worked out. The problem of optimal designing of temperature measurements has been solved in the course of the experiment. Data on the complex of thermophysical properties of the composite material have been obtained.

1. INTRODUCTION

Extended use of composite materials (glass, carbon and organic fiber-based composites) in rocket, aviation, automobile and other equipment demands detailed research of their characteristics. One of the most difficult problems is the problem of obtaining reliable data on the thermophysical properties of composite materials. The point is that by nature composite materials possess a pronounced anisotropy of properties and in addition, their properties depend upon the type of a semi-finished item of which the specimen is produced. The latter circumstance complicates the use of the hot guard plate method which is frequently used in thermophysical research, as very often one fails to produce a specimen with the required orientation and ratio of width and thickness of a concrete semi-finished item, e.g. a plate.

In this connection the use of complex identification methods of thermophysical properties of composite materials based on solving two-dimensional thermal conductivity inverse problems can be regarded as perspective. In this case the totality of the thermophysical properties of the material (volumetric heat capacity $C(T)$ and thermal conductivity in two directions $\lambda_x(T)$ and $\lambda_y(T)$) is determined from the data of one experiment.

2. EXPERIMENTAL FACILITY

At the Moscow Aviation Institute there has been developed an experimental module for carrying out thermophysical research on composite materials in the temperature range from 0 to 150 ^{0}C. In this temperature range no irreversible physicochemical changes take place in composite materials, i.e. the material can be regarded as thermostable. One should mention that the idea of the experimental scheme and the statement of the two-dimensional thermal conductivity coefficient inverse problem for the scheme are presented in [1].

The specimen for experimental research is gathered from 4 plates, sized 64×64 mm and 6 mm thick each, which are cut out of a plate blank. In order to prevent contact resistance between the plates their surface is covered with highly thermal conductive lubrication. The completed specimen is placed between two aluminium radiators that are cooled with flowing water. Two other sides of the specimen are pressed to massive aluminium plates which are in thermal contact with radiators and two last sides of the specimen are covered by heat insulation layers, see Figure 1.

In order to provide a good thermal contact of the specimen with the radiators and aluminium plates the highly thermal conductive lubrication is used. Experimental module of this kind permits a stable temperature close to the two-dimensional one in the central section of the specimen and provides a stable temperature on the border of the specimen.

Before the start of the experiment the specimen is kept in the experimental module with water switched on through the radiators for about 2 hours. It contributes to temperature evenness in the specimen.

The heating source in the process of the experiment is an electrical heater made of Nichrome alloy, 0.5 mm in diameter located in the mechanical center of the specimen. The capacity of the heater during the experiment is changed according to predetermined dependence.

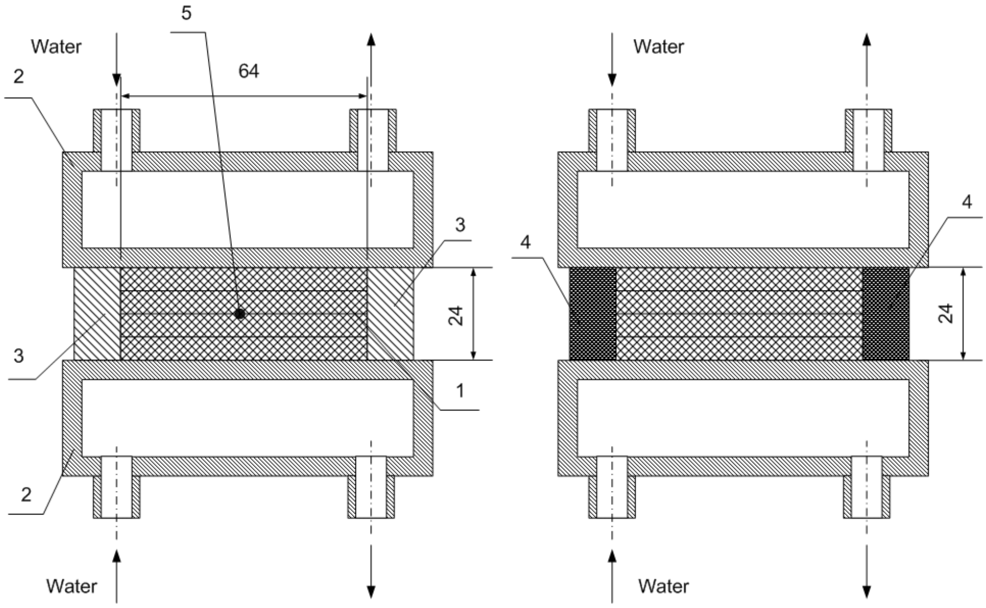

Figure1. Scheme of the experimental module 1-specimen; 2-water-cooled radiators; 3-aluminium plates; 4-heat insulation layings; 5 -heater.

3. MATHEMATICAL MODEL OF HEAT TRANSFER PROCESS

The mathematical model of two-dimensional unsteady heat transfer in the experimental module can be presented as follows:

$$C(T)\frac{\partial T(x,y,\tau)}{\partial \tau}=\frac{\partial}{\partial x}\left(\lambda_x(T)\frac{\partial T(x,y,\tau)}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda_y(T)\frac{\partial T(x,y,\tau)}{\partial y}\right)+S(x,y,\tau), \quad (1)$$

where: T the temperature; x, y the coordinates; τ the time; $C(T)$ the volumetric heat capacity of the material; $\lambda_x(T)$ and $\lambda_y(T)$ the thermal conductivity of the material in X and Y directions and $S(\tau,x,y)$ the centred heat source.

The temperature field of the specimen initially is constant, i.e.

$$T(x,y,0)=T_0 \,. \quad (2)$$

In the course of the test the temperature of its boundaries does not change, i.e.

$$T(x,y,\tau)=T_0 \text{, at } x=0 \text{ and } x=\Delta_x \text{, and at } y=0 \text{ and } y=\Delta_y \quad (3)$$

Heating of the specimen is performed with the help of a centred heating source located at the point with coordinates x_s and y_s. The power of the heating source changes through time in a predetermined way, namely:

$$S(x,y,\tau)=\delta(x-x_s)\cdot\delta(y-y_s)\cdot Q(\tau)\,, \quad (4)$$

where $Q(\tau)$ is the linear specific power of the electric heater.

In the course of the test temperature values are measured at a number of points of the specimen. One should determine the temperature dependences of the volumetric heat capacity $C(T)$ and the thermal conductivity $\lambda_x(T)$ and $\lambda_y(T)$ from these changes, i.e. solve a two-dimensional thermal conductivity inverse problem.

4. TWO-DIMENSIONAL THERMAL CONDUCTIVITY INVERSE PROBLEM

In order to solve the inverse problem for determining thermophysical properties we use an extreme statement [2], in which the temperature dependent anisotropic material thermal conductivity and the temperature dependent volumetric heat capacity, which give the minimum to the quadratic discrepancy functional of the experimental and calculated temperature values with respect to the discrepancy principle requirements, are defined as follows:

$$S(u)=\frac{1}{2}\int_0^{\tau_m}\sum_{n=1}^{N_t}\left(T\;\left(x_n^t,y_n^t,\tau\right)-T^e\left(x_n^t,y_n^t,\tau\right)\right)^2 d\tau; \quad u=\left\{\lambda_y(T),\lambda_x(T),C(T)\right\} \quad (5)$$

$$S(u)=\min;\;\; S\geq\Delta^2\,,$$

where τ_m is the duration of the experiment, N_t is the number of temperature sensors; Δ is the temperature measurement error; T^e are the experimental temperature values; and x_n^t and y_n^t are the locations of the temperature sensors.

The required temperature dependent thermal properties are represented in the following parametric way:

$$\lambda_x(T)=\sum_{i=1}^{K_{\lambda x}}\lambda_{xi}\varphi_{1i}(T);\quad \lambda_y(T)=\sum_{j=1}^{K_{\lambda y}}\lambda_{yj}\varphi_{2j}(T);\quad C(T)=\sum_{k=1}^{Kc}C_k\varphi_{3k}(T), \tag{6}$$

where φ_{1k}, φ_{2k} and φ_{3k} are the basic functions; $K_{\lambda x}, K_{\lambda y}, K_C$ are the number of parameters used to represent the unknown temperature dependent thermophysical properties $\lambda_x(T), \lambda_y(T), C(T)$.

The inverse solution is based on an iterative regularization method in which the termination of the iterative process is performed according to the discrepancy principle. The problem of functional minimization (5) is solved by the method of conjugate gradients. The iterative minimization process can be presented as follows:

$$u^{n+1}=u^n-\gamma_n\cdot s^n; \tag{7}$$

$$s^{n+1}=G'^{n+1}+\beta^n\cdot G'^n; \tag{8}$$

$$\beta^0=0; \tag{9}$$

$$\beta^n=\frac{(G'^n,G'^{n+1}-G'^n)}{\left\|G'^n\right\|^2}, \tag{10}$$

where the superscript n denotes the iteration number.

The gradient of the residual functional G' is defined according to the parameters of the sought parameters based on the adjoint problem solution. The adjoint problem is as follows:

$$\begin{aligned}-C(T)\frac{\partial\,\psi(x,y,\tau)}{\partial\,\tau}&=\lambda_x(T)\frac{\partial}{\partial\,x}\left(\frac{\partial\,\psi(x,y,\tau)}{\partial\,x}\right)+\lambda_y\,(T)\frac{\partial}{\partial\,y}\left(\frac{\partial\,\psi(x,y,\tau)}{\partial\,y}\right)+\\&+\sum_{n=1}^{N_t}\left(T\ \left(x_n^t,y_n^t,\tau\right)-T^e\left(x_n^t,y_n^t,\tau\right)\right)\cdot\delta(x-x_n^t)\cdot\delta(y-y_n^t),\end{aligned} \tag{11}$$

$$x\in[0,\Delta_x];\quad y\in\left[0,\Delta_y\right];\ \tau\in[0,\tau_m];$$

$$\tau=\tau_m\qquad \psi(x,y,\tau_m)=0; \tag{12}$$

$$x=0\ \text{ and }\ x=\Delta_x;\ y=0\ \text{ and }\ y=\Delta_y\qquad \psi(x,y,\tau)=0. \tag{13}$$

The gradient of the functional is found as follows:

$$G'_{\lambda_{xi}}=\int_0^{\tau_m}\int_0^{\Delta_x}\int_0^{\Delta_y}\frac{dT}{dx}\frac{d\psi(x,y,\tau)}{dx}\varphi_{1i}(T)\ dxdyd\tau;\qquad i=\overline{1,K_{\lambda x}}; \tag{14}$$

$$G'_{\lambda_{yj}}=\int_0^{\tau_m}\int_0^{\Delta_x}\int_0^{\Delta_y}\frac{dT}{dy}\frac{d\psi(x,y,\tau)}{dy}\varphi_{2j}(T)\ dxdyd\tau;\qquad j=\overline{1,K_{\lambda y}}; \tag{15}$$

$$G'_{C_k}=\int_0^{\tau_m}\int_0^{\Delta_x}\int_0^{\Delta_y}\frac{dT}{dt}\psi(x,y,\tau)\varphi_{3k}(T)\,dxdyd\tau;\qquad k=\overline{1,K_C}; \tag{16}$$

$$G'=\{G'_{\lambda_i},G'_{\lambda_j},G'_{C_k}\};\qquad i=\overline{1,K_{\lambda x}};\quad j=\overline{1,K_{\lambda y}};\quad k=\overline{1,K_C}. \tag{17}$$

The iteration step is determined from the following expression:

$$\gamma^n=\frac{\sum_{k=1}^{N_t}\int_0^{\tau_m}v(x_k^t,y_k^t,\tau,u^n)\cdot\left(T\ \left(x_k^t,y_k^t,\tau\right)-T^e\left(x_k^t,y_k^t,\tau\right)\right)d\tau}{\sum_{k=1}^{N_t}\int_0^{\tau_m}v(x_k^t,y_k^t,\tau,u^n)^2d\tau}. \tag{18}$$

The sensitivity functions $v(x,y,\tau)$ are determined by solving the following problem:

$$\begin{aligned}\frac{\partial\,C(T)\,v(x,y,\tau)}{\partial\,\tau}&=\frac{\partial}{\partial\,x}\left(\frac{\partial\,\lambda_x(T)\ v(x,y,\tau)}{\partial\,x}\right)+\frac{\partial}{\partial\,y}\left(\frac{\partial\,\lambda_y\,(T)\ v(x,y,\tau)}{\partial\,y}\right)+\\&+\frac{\partial}{\partial\,x}\left(\delta\lambda_x\ \frac{\partial\,T(x,y,\tau)}{\partial\,x}\right)+\frac{\partial}{\partial\,y}\left(\delta\lambda_y\ \frac{\partial\,T(x,y,\tau)}{\partial\,y}\right)-\delta C(T)\frac{\partial\,T(x,y,\tau)}{\partial\,\tau},\end{aligned} \tag{19}$$

$$x\in[0,\Delta_x];\quad y\in\left[0,\Delta_y\right];\ \tau\in[0,\tau_m];$$

$$\tau=0\qquad v(x,y,0)=0; \tag{20}$$

$x = 0$ and $x = \Delta_x$; $y = 0$ and $y = \Delta_y$ $\qquad v(x, y, \tau) = 0;$ (21)

$\delta u = \{\delta\lambda_x, \delta\lambda_y, \delta C\}$, $\delta u = s^n$.

5. RESULTS OF MATHEMATICAL MODELLING

The accuracy and the stability of the inverse problem solution were investigated by the method of computational modeling. At the first stage there were calculated temperature values in the presupposed location of the sensors and at the second stage these "experimental" temperatures were used for solving the inverse problem.

In Figure 2 there are presented the results of the two-dimensional inverse problem of determining the thermophysical properties of composite material for different numbers (1, 2, 4) of linear approximation of the sought temperature dependences. It is obvious that with a growing number of approximation intervals, the solution accuracy is reduced and the solution becomes oscillatory.

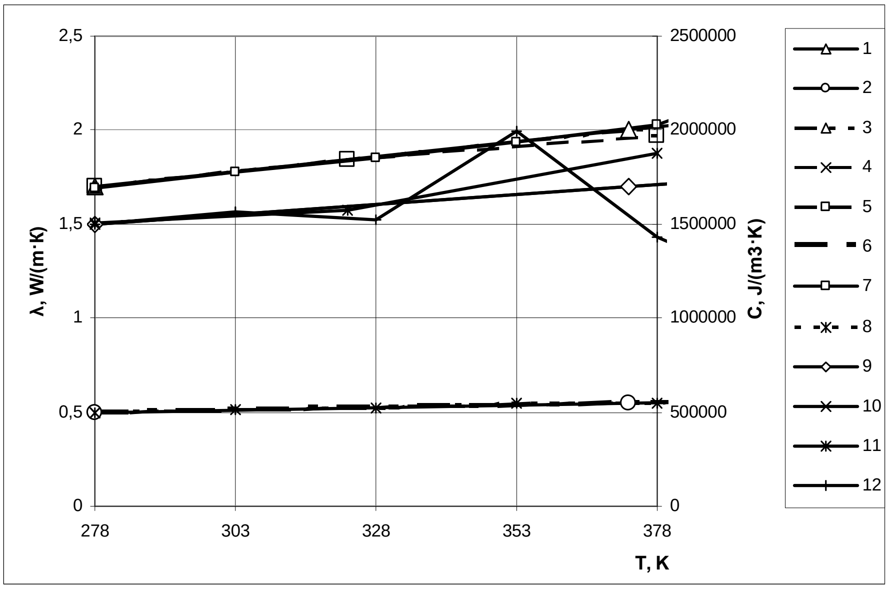

Figure 2. Solution of a two-dimensional inverse problem for various numbers of approximation regions of the temperature dependences for the thermophysical properties.
1, 2, 9 - $\lambda_x(T)$, $\lambda_y(T)$, $C(T)$ - of the model material;
3, 4, 10 – solution results for 1 interval;
5, 6, 11 – solution results for 2 intervals;
7, 8, 12 – solution results for 3 intervals.

Figure 3 illustrates the inverse problem solution for 5% "noisy" data introduced in the input temperature data. It is clear that more considerable solution errors, just like in the previous case, are observed by the use of a greater number of linear approximation intervals.

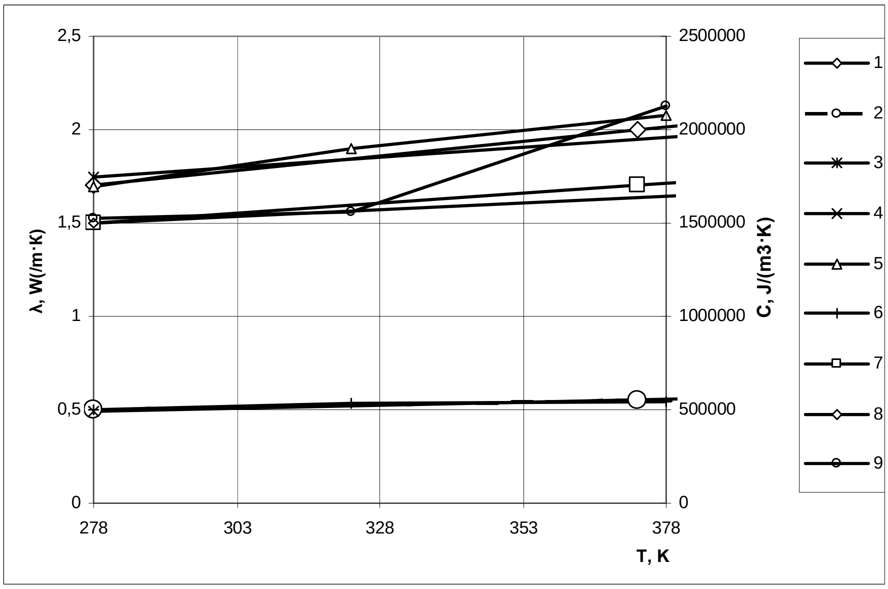

Figure 3. Solution of a two-dimensional inverse problem for various numbers of approximation regions of temperature dependences for the thermophysical properties. Case of noisy data.
1, 2, 7 - $\lambda_x(T)$, $\lambda_y(T)$, $C(T)$ - of model material;
3, 4, 8 – solution results for 1 interval;
5, 6, 9 – solution results for 2 intervals.

6. STATEMENT OF THE TEMPERATURE MEASUREMENT DESIGN PROBLEM

While organizing thermophysical research, the choice of rational temperature sensors location is an important question. The general approach to the problem of temperature measurement design while determining thermal conductivity is given in [2, 3]. In the present research this approach is used for the optimal design of temperature measurements while solving the two-dimensional thermal conductivity coefficient inverse problem.

Under the measurement design $\Lambda = \{N_t, (x_1, y_1), (x_2, y_2), (x_3, y_3) \ldots (x_{N_t}, y_{N_t})\}$, we understand the totality of N_t number and location of temperature sensors which contribute to the best accuracy of inverse problem solution. For the quality measure of the measurement design we take the determinant of the normalized Fisher matrix F. Elements of Fisher matrix are determined as follows:

$$F(\Lambda) = \frac{1}{N_t}\Phi_{jk}, \quad j,k = \overline{1,K},$$

where $K = K_{\lambda_x} + K_{\lambda_y} + K_C$ and $\Phi_{jk} = \sum_{n=1}^{N_t}\int_0^{\tau_m}\theta_j(x_n, y_n, \tau)\cdot\theta_k(x_n, y_n, \tau)\,d\tau$. Values of $\theta_k(x_n, y_n, \tau)$ are sensitivity functions of the temperature field at the point of the temperature sensor location. The sensitivity functions are determined from the solution of the following boundary-value problem:

$$\frac{\partial\, C(T)\,\theta_k(x,y,\tau)}{\partial\,\tau} = \frac{\partial}{\partial\,x}\left(\frac{\partial\,\lambda_x(T)\,\theta_k(x,y,\tau)}{\partial\,x}\right) + \frac{\partial}{\partial\,y}\left(\frac{\partial\,\lambda_y\,(T)\;\theta_k(x,y,\tau)}{\partial\,y}\right) + Q_k, \tag{22}$$

$$x \in [0, \Delta_x]; \quad y \in [0, \Delta_y]; \; \tau \in [0, \tau_m]; \quad k = \overline{1, K_C + K_{\lambda x} + K_{\lambda y}},$$

$$\tau = 0; \qquad \theta_k(x, y, 0) = 0; \tag{23}$$

$$x = 0 \text{ and } x = \Delta_x; \; y = 0 \text{ and } y = \Delta_y; \qquad \theta_k(x, y, \tau) = 0; \tag{24}$$

$$Q_k = \begin{cases} \dfrac{\partial}{\partial\,x}\left(\varphi_{1k}\,\dfrac{\partial\,T(x,y,\tau)}{\partial\,x}\right), \; k = \overline{1, K_{\lambda x}}, \\ \dfrac{\partial}{\partial\,y}\left(\varphi_{2k}\,\dfrac{\partial\,T(x,y,\tau)}{\partial\,y}\right), \; k = \overline{1 + K_{\lambda x}, K_{\lambda x} + K_{\lambda y}}, \\ -\varphi_{3k}\,\dfrac{\partial\,T(x,y,\tau)}{\partial\,\tau}, \; k = \overline{1 + K_{\lambda x} + K_{\lambda y}, K_C + K_{\lambda x} + K_{\lambda y}} \end{cases} \tag{25}$$

In order to find the Fisher matrix determinant, the famous non-gradient Nelder-Mead method is used.

7. SOLUTION OF THE MEASUREMENT DESIGN PROBLEM

For the conditions that occur in the experimental module, see Figure 1, a problem of designing temperature measurements was solved. If one does not place restrictions on the location of temperature sensors then the measurement plan in Figure 4 result 1 is the optimal one. However, the location of a temperature sensor at the position where the heater is located or close to it (at a distance of less than 1 mm) is impossible. In addition the specimen is constructed from separate plates 6 mm thick each and damaging the plates for the installation of the temperature sensors is undesirable. That is why one should reduce the permissible measurement design by demanding that temperature sensors are located on the boundaries of the plates and at a distance not less than 1 mm from the heater. Coordinates of temperature sensors at optimal designing with these restrictions are also presented in Figure 4 result 2.

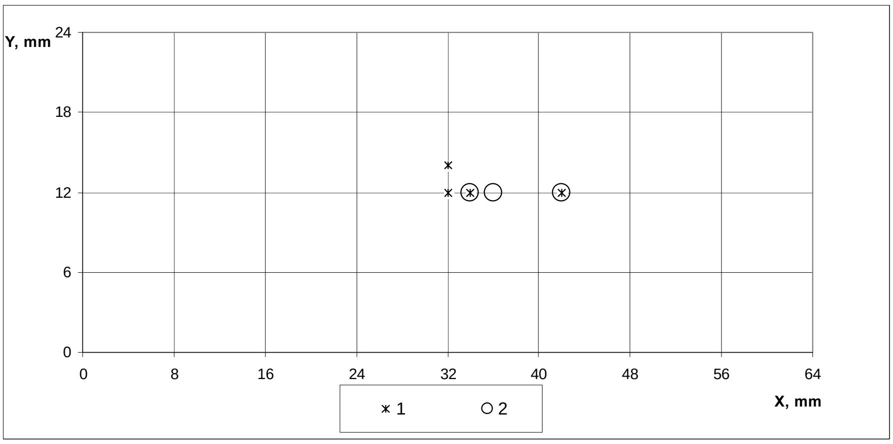

Figure 4. Solution of the problem of temperature measurements optimal designing without any restrictions (1) and with restricted temperature sensors location (2).

8. EXPERIMENTAL RESEARCH AND DATA PROCESSING

Experimental research was carried out at Moscow Aviation Institute by engineers Klimenko and Razuvanov. As temperature sensors there were used thermocouples with thermoelectrodes 0.2 mm in diameter located according to the measurement design, see Figure 4. Thermocouples and the heater were laid into special riffles one after another, then they were stretched and fixed to the plates with the help of special adhesive. Thermoelectrodes of the thermocouples and the heater were insulated from the material of the specimen by special covering. Figure 5 illustrates the dependence of specific power changes of the electrical heater in time and the indications of the thermocouples.

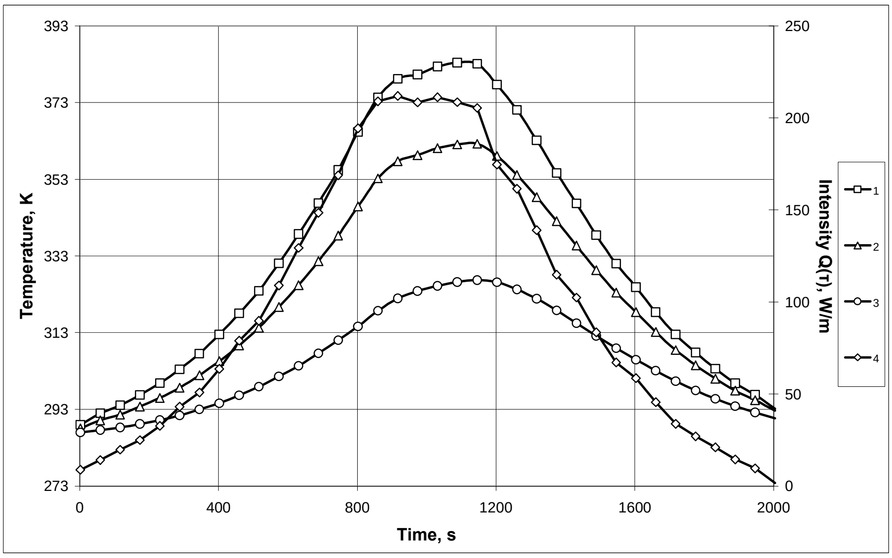

Figure 5. Indications of temperature sensors, installed at the following points
1- x =34 mm; y =12mm,
2- x =36 mm; y =12 mm,
3- x =42 mm; y =12 mm
and dependence of the heater specific power during the experiment (4).

Obtained experimental data were used as input data while solving a two-dimensional inverse problem. For the numerical solution of the inverse problem a finite-element fragmentation of the calculation area with a step of 1.0 mm was used. The step in time was taken to be 5 seconds. The nonlinear direct problem was solved iteratively, using Newton's iteration, with an accuracy of 0.01 degree. For solving the two-dimensional inverse problem the conjugate gradient method with a vector choice of step was used. To guarantee uniqueness of the solution a series of computations for different choices of initial approximation was performed. The initial approximation of the volumetric heat capacity varied in the range of 1000000-2500000 J/m^{3}K while the initial approximation of the thermal conductivity was 0.5-2.5 W/mK. The iterative process terminated on the condition of coincidence of the results of two consequent iterations with accuracy of 0.5%. Solutions of the two-dimensional inverse problem of identifying thermophysical properties of the composite material are presented in Figure 6.

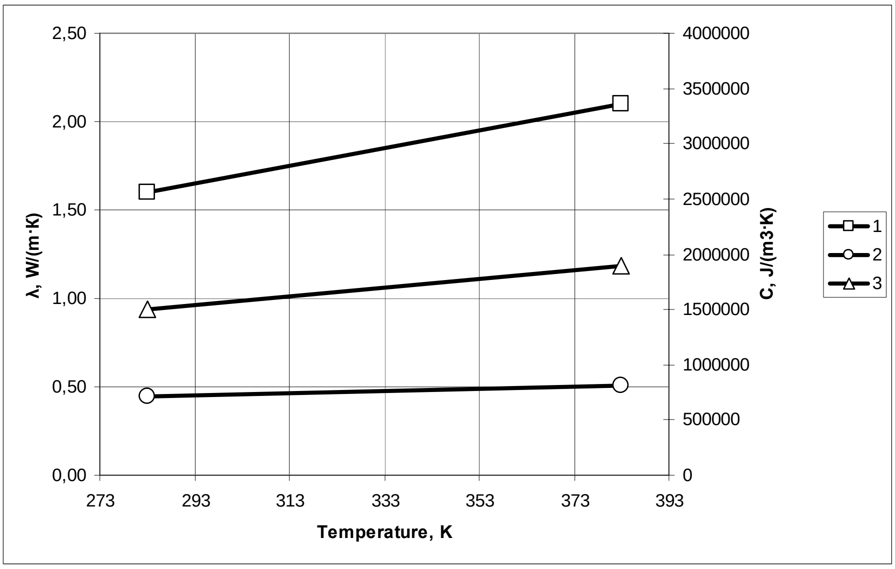

Figure 6. Solution of a two-dimensional inverse problem of determining the thermophysical properties of a composite material.
1 - $\lambda_x(T)$; 2 - $\lambda_y(T)$;
3 - $C(T)$

9. ANALYSIS OF RESULTS

In order to control the solution of the two-dimensional inverse problem a comparison was performed between calculated values of temperature obtained with the use of temperature dependences of thermophysical properties of the material under consideration, determined from the inverse problem solution with experimental temperature values. In Figure 7 time dependences of difference between calculated and experimental temperature values are presented. One can notice that the maximal difference of calculated and experimental temperature values is not more than 2.7 K. It confirms the reliability of results obtained on finding thermophysical properties of the material.

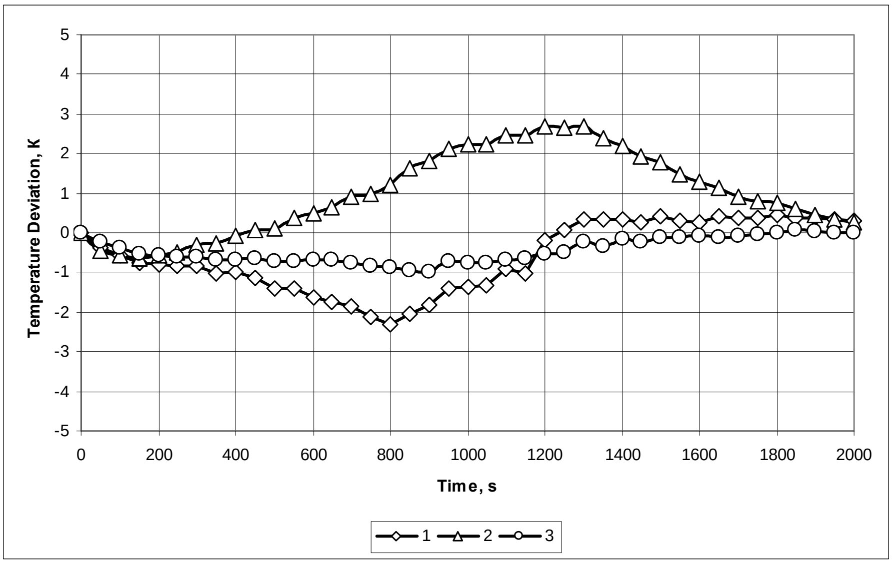

Figure 7. Differences of calculated and experimental temperature values. Numbers of curves coincide with these of thermocouples in Figure 5.

10. CONCLUSIONS

A solution algorithm of the two-dimensional thermal conductivity inverse problem for the determination of the thermophysical properties of anisotropic composite material has been worked out. An experimental module for carrying out thermophysical research has been created. Designing temperature measurements has been executed and regimes for carrying out experimental research have been chosen. A series of experiments has been undertaken and the thermophysical properties of composite material have been determined.

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